Polarization structure of combined vortex beams

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ABSTRACT

Nongeneric polarization structures of the vortex beams resulting from coherent coaxial mixing of orthogonally polarized one-charged Laguerre-Gaussian modes with different mode numbers are analyzed. General solution is derived for a superposition of elliptically orthogonally polarized partial vortex beams, and the limiting partial cases when the mixed modes are polarized linearly or circularly are explored both theoretically and experimentally. It is established that in such combined beams unusual spatially stable polarization structures arise, such as *closed* $C-$ contours and $L-$ contours with a *constant* azimuth of linear polarization.

Keywords: singular optics, vortex beams, polarization singularities, nongeneric structures

1. INTRODUCTION

Polarization singularities arising in spatially inhomogeneously polarized light fields, such as developed random speckle-fields, are among the most interesting subjects of study into Singular Optics[1]. Singular skeleton of such field is formed by the wave front disclinations as well as by $C-$points (points, where the field is circularly polarized) and $L-$contours (lines, where the field is linearly polarized) observed at the plane perpendicular to the mean direction of propagation of the paraxial optical beam[2]. Disclinations of light field are unobservable due to changes of a phase of the field with optical frequency. That is why, the main attention is paid to stable $C-$ and $^{\rho}/_{w_2}=1$singularities, as well as to the associated Stokes singularities[3]. In the case of *generic*[2] speckle-field, $C-$singularities may be only isolated (point-like), while any displacement from $C-$point leads unavoidably to violation of one of two (or of both) conditions of circular polarization, such as equal amplitudes of arbitrary linear field components and a quarter-wave shift among them. Azimuth of linear polarization at $L-$ contours also changes due to changing of the amplitude ratio of in-phase (or of out-of-phase by π) orthogonal components of the field along $L-$contour. These properties of polarization singularities, however, are not universal. Situation changes considerably, when one operates with deterministic vortex beams with controlled parameters.

In this paper, we analyze *nongeneric* polarization structure of the vortex beams resulting from coherent coaxial mixing of orthogonally polarized Laguerre-Gaussian (LG) modes with the same topological charges but with different mode numbers. In spite of nongenericity of polarization singularities at such beams, these artificially constructed structures may be useful in solving of some problems of optical telecommunications, optical computing, and optical nondestructive diagnostics owing to flexible control of them. In Section 2, mixing of two one-charged doughnut LG modes with the lowest numbers is considered. General solution is derived, which describes a superposition of elliptically orthogonally polarized partial vortex beams, and the limiting partial cases when the mixed beams are polarized linearly or circularly are explored. It is established that in this case unusual polarization structures arise, such as *closed* $C-$contours and $L-$contours with a *constant* azimuth of linear polarization. In Section 3, the experimental results are presented demonstrating such polarization structures into combined vortex beams. In Section 4, we summarize the main results of this study and very concisely discuss an analogy of the studied here polarization structures to the singularities of the cross-spectral density function of scalar partially coherent vortex beams with a separable phase[4].

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2. STABLE POLARIZATION STRUCTURES IN COMBINED VORTEX BEAMS

2.1. Construction of general solution

The simplest nontrivial case of the problem of interest consists in coherent coaxial mixing of orthogonally polarized LG modes LG_0^1 and LG_1^1. Following to the results of papers[4,5], we choose the modes with equal caustic parameters, equal signs of the topological charge, and with ratio of integral powers as $P_0/P_1 = 1.45$. Similarly to the case of incoherent superposition of such modes[4], there is no interference among orthogonally polarized modes, and the radial intensity distribution into combined beam resembles such distribution in a simple mode LG_0^1 alone, as in can be seen from Fig.1.

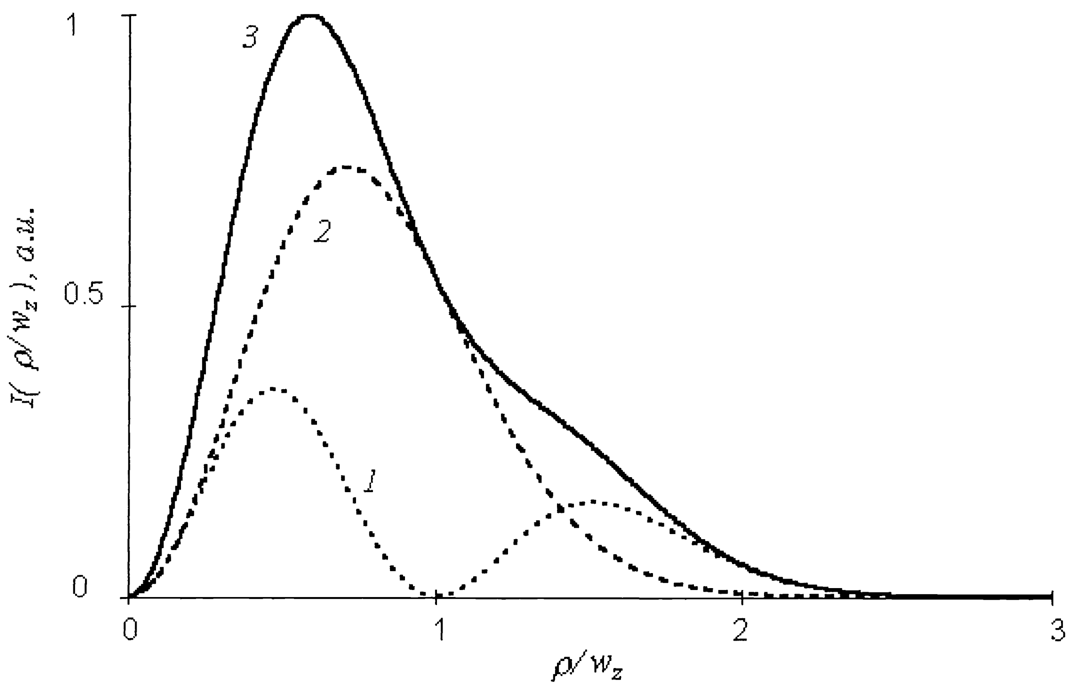

Fig. 1. Radial intensity distributions of orthogonally polarized modes LG_1^1 and LG_0^1 (curves 1 and 2, respectively) and of the combined singular beam (curve 3) for the ratio of integral powers of partial modes as 0.45:1.

Let us define the states of polarization of orthogonally polarized modes LG_0^1 and LG_1^1 by Jones vectors[6]

$$\begin{bmatrix} ae^{i\varphi_x} \\ be^{i\varphi_y} \end{bmatrix} \text{ and } \begin{bmatrix} -be^{-i\varphi_y} \\ ae^{-i\varphi_x} \end{bmatrix}, \tag{1}$$

respectively. Jones vector of the combined beam resulting from coherent coaxial mixing of partial modes is of the form:

$$\begin{bmatrix} a \\ be^{i\Phi_{yx}} \end{bmatrix} + ce^{-i\Delta} \begin{bmatrix} b \\ -ae^{i\Phi_{yx}} \end{bmatrix} = \begin{bmatrix} a + cbe^{-i\Delta} \\ b - cae^{-i\Delta}e^{i\Phi_{yx}} \end{bmatrix}. \tag{2}$$

Writing Eq. (1), we choose Cartesian coordinates (x, y) in such a way that initially the $x-$components of partial modes are in-phase in the vicinity of the central optical vortex being in opposite phases at $\rho/w_z > 1$, where the phase of the mode LG_1^1 changes by π. In Eq. (2), $\Phi_{yx} = \varphi_y - \varphi_x$; $c \equiv c(\rho/w_z)$ is the ratio of the mode amplitudes *vs* dimensionless parameter ρ/w_z that is equal to the ratio of the radial coordinate $\rho = \sqrt{x^2 + y^2}$ to the beam width w_z at distance z from the caustics waist (hereinafter the functional dependence of $c(\rho/w)$ is suppressed for readability); Δ is the phase shift among the modes, which can be assumed to be constant at the combined beam cross-section in supposition of equal caustics parameters and, consequently, equal curvature of the mixed wave fronts at the observation plane that is equidistant from two caustics planes.

The beam with the Jones vector (2) is described by the following Stokes parameters:

$$\begin{aligned}
S_0 &= \left(1+c^2\right)\left(a^2+b^2\right),\\
S_1 &= \left(1-c^2\right)\left(a^2-b^2\right)+4cab\cos\Delta,\\
S_2 &= 2\left\{\left(1-c^2\right)ab\cos\Phi_{yz} - c\left[a^2\cos\left(\Delta-\Phi_{yx}\right)-b^2\cos\left(\Delta+\Phi_{yx}\right)\right]\right\},\\
S_3 &= 2\left\{\left(1-c^2\right)ab\sin\Phi_{yx} + c\left[a^2\sin\left(\Delta-\Phi_{yx}\right)+b^2\sin\left(\Delta+\Phi_{yx}\right)\right]\right\}.
\end{aligned} \tag{3}$$

It is seen from Eqs. (3) and from Fig. 1 that *all* Stokes parameters of the combined beam are the functions of the radial coordinate. Thus, the combined beams occur to be spatially inhomogeneously polarized, and both the intensity distribution and the polarization structure of such beams possess axial symmetry. From Eqs. (3) one can find the polarization azimuth, $\alpha = \frac{1}{2}\arctan\frac{S_2}{S_1}$, and the ellipticity angle, $\beta = \frac{1}{2}\arcsin S_3$. Eqs. (3) are quite general, being applied to the analysis of the combined beams formed by arbitrary orthogonally, in general elliptically, polarized modes. Below we analyze the most pronouncing limiting cases, when the partial modes are linearly or circularly polarized.

2.2. Limiting partial cases

2.2.1. $b = 0$ - partial modes are linearly polarized with azimuths 0^0 and 90^0.

Due to arbitrary choice of Cartesian basis (x, y) for decomposition of the field into orthogonal linearly polarized components, the considered case is rather general one.

For $\Delta = \pm\pi/2$, one finds from Eqs. (3) the normalized second, third, and fourth Stokes parameters:

$$\left\{\frac{1-c^2}{1+c^2};\ 0;\ \pm\frac{2c}{1+c^2}\right\}. \tag{4}$$

Eq. (4) describes walking of the point imaging the state of polarization of the combined singular beam at Poincare sphere along the meridian, which is determined by the states of polarization of partial modes. Thus, the combined beam is generally characterized by elliptical states of polarization with the ellipticity angle $\beta = \frac{1}{2}\arcsin\left(\pm\frac{2c}{1+c^2}\right)$. The radial distribution of the ellipticity angles into combined beam is shown in Fig. 2.

The polarization azimuth $\alpha = 0^\circ$ at $\rho/w_z < 1.45$, where $|c| < 1$, and $\alpha = 90^\circ$ at $\rho/w_z > 1.45$, where $|c| > 1$. At the ring $\rho/w_z = 1$, where $c = 0$, the beam is linearly polarized, $\alpha = 0^\circ$; the angle of ellipticity is undetermined, while the argument of $\arcsin$ changes its sign at this ring. At the ring $\rho/w_z \approx 1.45$, where $|c| = 1$, the beam is left-hand or right-hand circularly polarized when $\Delta = +\pi/2$ or $\Delta = -\pi/2$, respectively (c is negative!). This state of polarization is imaged at the pole of Poincare sphere, where the polarization azimuth is undetermined due to indetermination of $\arctan(0/0)$. Note, in terms of Stokes singularities[3], this situation corresponds to the singularity of the complex function $S_{12} = S_1 + iS_2$.

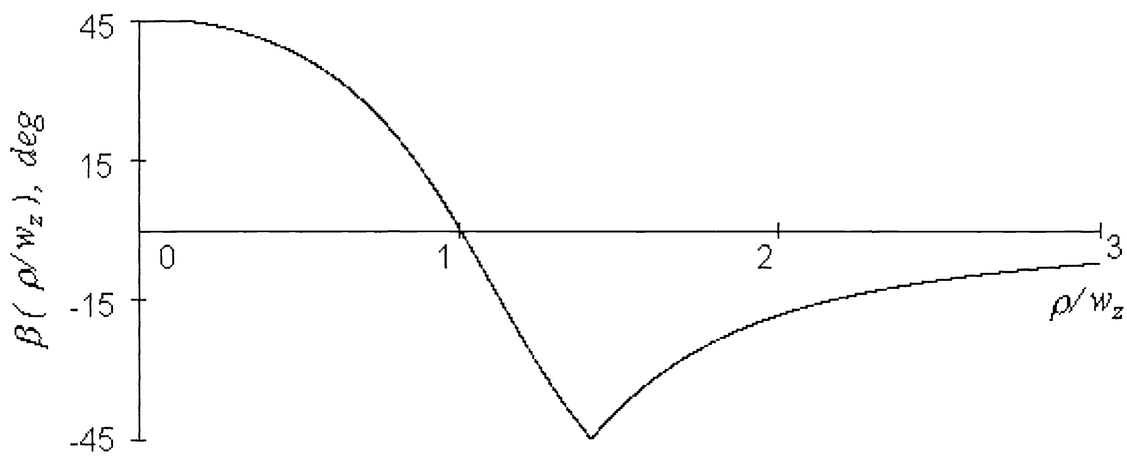

Fig. 2. Radial distribution of the angle of ellipticity at the combined singular beam resulting from coherent mixing of orthogonally linearly polarized modes with a phase difference $\Delta = \pi/2$.

Thus, $L-$ *contour with the constant azimuth of linear polarization* $\alpha = 0^\circ$ (at $\rho/w_z = 1$) and *closed* $C-$ *contour* (at $\rho/w_z \approx 1.45$) are realized at the field of elliptic states of polarization.

2.2.2. $b = a, \Phi_{yx} = \pm\pi/2$ *- partial modes are orthogonally circularly polarized.*

From Eqs. (3), one finds the normalized second, third and fourth Stokes parameters:

$$\left\{\frac{2c}{1+c^2}\cos\Delta;\ \mp\frac{2c}{1+c^2}\sin\Delta;\ \frac{1-c^2}{1+c^2}\right\}, \tag{5}$$

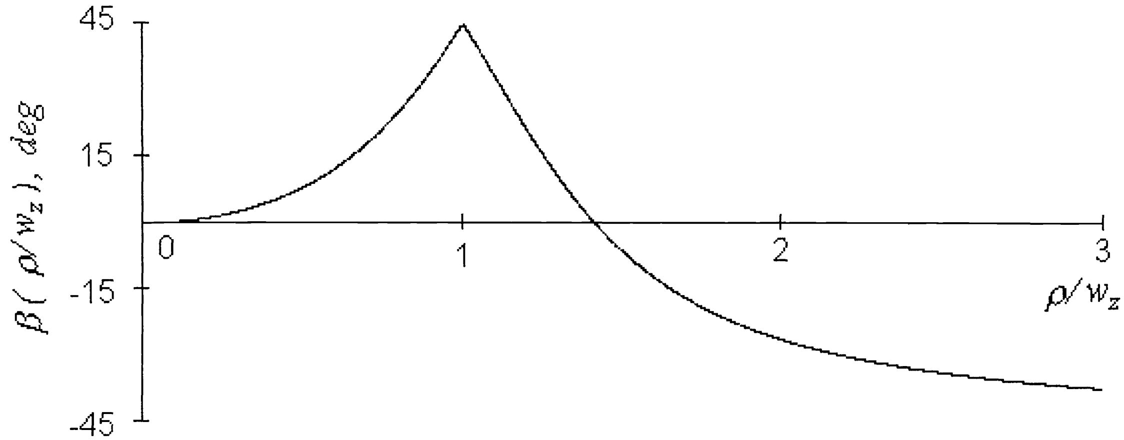

Fig. 3. Radial distribution of the angle of ellipticity at the combined singular beam resulting from coherent mixing of orthogonally circularly polarized modes.

polarization azimuth $\alpha = \mp \Delta/2$, and ellipticity angle $\beta = \frac{1}{2}\arcsin\frac{1-c^2}{1+c^2}$. Eq. (5) describes walking of the point imaging the state of polarization of the combined singular beam at Poincare sphere along the meridian, which is determined by the phase difference of the partial modes, Δ. Again, spatial distribution of elliptic states of polarization is generally realized into combined beam. Radial distribution of the angles of ellipticity for this case is shown in Fig. 3. At the ring $\rho/w_z = 1$, where $c = 0$, azimuth is undetermined due to indeterminicity of $\arctan(0/0)$, and $\beta = \pm\pi/4$; one obtains here $C-$contour with the state of polarization of the mode LG_1^1. At the ring $\rho/w_z = 1.45$, where $|c| = 1$, $\beta = 0^\circ$ (for the same polarization azimuth $\alpha = \mp\Delta/2$); one obtains the ring singularity of the field of ellipticity as $L-$contour with the constant azimuth of linear polarization. Crossing this ring, the imaging point at Poincare sphere passes from north hemisphere to south one or vise versa (the fourth Stokes parameter changes its sign) due to prevailing the mode LG_1^1 at $\rho/w_z > 1.45$.

3. EXPERIMENT

3.1. Technique for revealing of polarization singularities in combined vortex beams

Convenient experimental technique for revealing of nongeneric polarization structures into combined vortex beams can be realized using the generalized Malus law formulated into terms of Stokes parameters. The *own polarization vector* can be associated with a polarization analyzer (in general, of elliptical type)[6], i.e. the vector originating from the center of the Poincare sphere and ending at the point of this sphere that images the state of polarization of the beam, which passes the analyzer with maximal intensity. In the same manner, one can define the polarization vector of the tested beam as the vector from the center of Poincare sphere to the imaging point at the sphere. Choosing the radius of the Poincare sphere to be unitary, one can express the coordinates of both vectors by the corresponding *normalized* Stokes parameters of the beam, $\bar{s}_i^{(B)}$, and of the analyzer, $\bar{s}_i^{(A)}$, where $\bar{s}_i = S_i/S_0$, $i = 0,1,2,3$. Then, optical transmittance of a polarization analizer, τ, is represented as

$$\tau \sim \cos^2\theta = \frac{1}{2}\sum_{i=0}^{3}\bar{s}_i^{(B)}\bar{s}_i^{(A)}\,, \tag{6}$$

where θ is a half-angle between the own vector of the polarization analyzer (A) and the polarization vector of the tested beam (B) at the Poincare sphere.

An example of applying of this technique is illustrated in Fig. 4. Curve 1 in Fig. 4 corresponds to the case 2.2.1 with $\Delta = +\pi/2$; the own vector of circular polarization analyzer corresponds to right-hand polarization. In this case, one observes the radial intensity distribution behind the analyzer, which resembles such distribution at partial mode LG_1^1, but with zero at the ring $\rho/w_z \approx 1.45$ rather than at the ring $\rho/w_z = 1$. At this ring one detects (by polarization analysis) a ring singularity of the field of the angles of ellipticity. Curves 2 and 3 at Fig. 4 correspond to the beams at the output of a polarization analyzer, which support the central vortex alone, while the ring singularities of polarization parameters are absent. Curve 2 characterizes the combined beam produced by two orthogonally linearly polarized modes with quarter-period phase shift passes the left circular polarization analyzer. Curve 3 corresponds to a simple case, when one observes only homogeneous attenuation of the beam shown by curve 3 at Fig. 1. This result takes place, when the combined beam produced by two orthogonally linearly polarized modes with quarter-period phase shift passes the linear polarization analyzer whose axis of maximal transmittance makes the angles $\pm 45^\circ$ with polarization azimuths of the constituting modes.

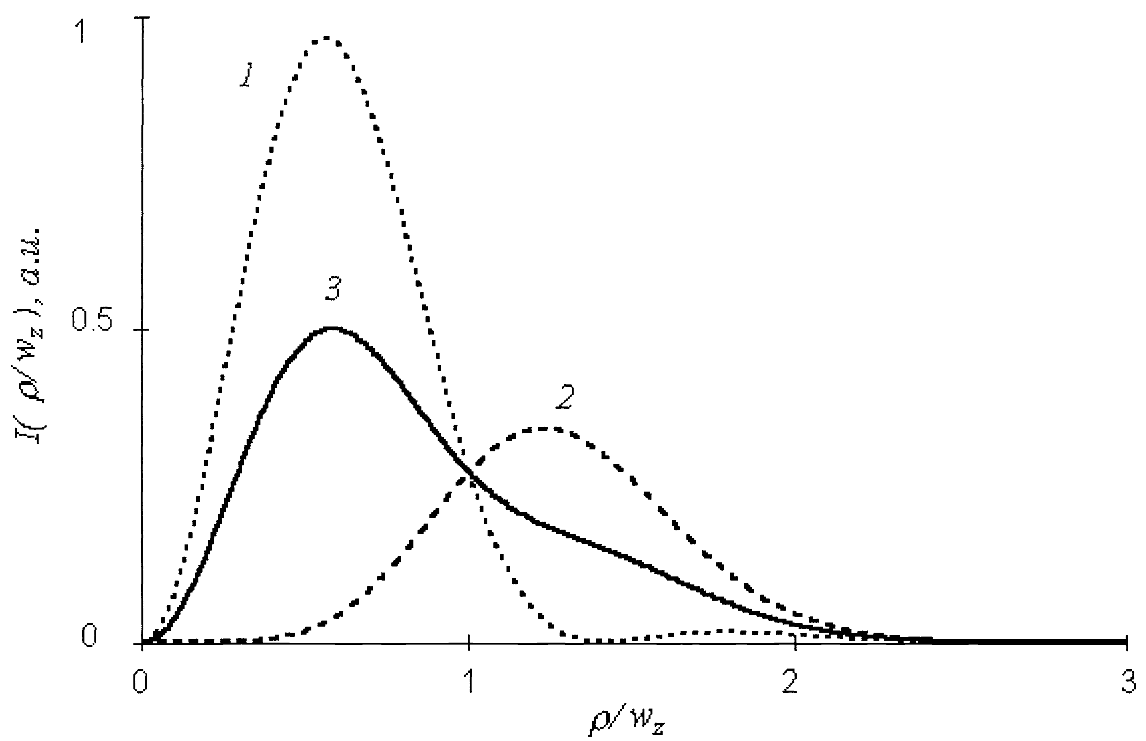

Fig. 4. Radial intensity distributions at the output of polarization analyzer of the combined vortex beam with intensity distribution shown in Fig. 1, curve 3.

The presence of the ring singularities of polarization parameters (as well as of the central optical vortex) in combined, spatially inhomogeneously polarized singular beams can be verified by testing of the beam at the output of a polarization analyzer using an opaque strip placed symmetrically in respect to the center of the beam. This experimental technique has been introduced in[4] for testing of partially spatially coherent singular beams. The central vortex is diagnosed by typical bending of interference fringes behind the strip, whose magnitude and direction characterize unambiguously the module and the sign of the topological charge of an optical vortex. Ring singularity (in[4] – ring singularity of the complex amplitude at the mode LG_1^1) is revealed from a half-period shift of interference fringes in the vicinity of the line of zero amplitude (at the ring $\rho/w_z = 1$). This technique seems to be also preferable into the problem considered here, while it permits to avoid the difficulties following from the difference of the states of polarization of the referent beam and of the *spatially inhomogeneously* polarized tested singular beam, when one uses a classical interferometric technique for detecting of optical singularities[1].

3.2. Experimental results

Experiment on generation of the combined singular beams by a coherent mixing of orthogonal in polarization modes LG_0^1 and LG_1^1 and testing of polarization structure of such beams has been performed in the arrangement of Mach-Zehnder interferometer, Fig. 5. He-Ne laser L emits a linearly (vertically) polarized beam LG_0^0 with the following parameters: $\lambda = 0.6328\ \mu m$, power $\sim 30\ mW$, the caustics waist width $2w_0 = 1.6\ mm$, diffraction spreading $\theta \approx 2.5 \times 10^{-4}\ rad$, Rayleigh distance $z_R \approx 3.177\ m$. For these parameters of the beam, at distance $z = 1\ m$ from the caustics the radius of curvature of a wave front is $R \approx 11\ m$, and $2w_z = 1.677\ mm$.

A beam splitter BS_1 splits the beam in two beams of equal amplitudes. Off-axis computer-generated holograms, CGH_1 and CGH_2, computed to generate the modes LG_0^{-1} and LG_1^{-1}, respectively, in the first diffraction orders are placed into the legs of the interferometer. As it is known [1, 4], both beams support central one-charged optical vortices, and, besides, the mode LG_1^{-1} supports the ring singularity. Spatial filters SF_1 and SF_2 are placed behind the holograms for selection of the actual diffraction orders.

The states of polarization of the modes mixed at the beam-splitting cube BS_2 are controlled in the following way. A quarter-wave plate is placed at the interferometer input. If any axis of this plate coincides with the azimuth of polarization of the laser beam, then both holograms are illuminated by equally linearly polarized beams. Polarization orthogonality of the modes LG_0^{-1} and LG_1^{-1} is provided by the use of a half-wave plate, whose fast axis makes the angle $\pm 45^\circ$ with the polarization azimuth of the laser beam. In our experiment, a half-wave plate is introduced into the mode LG_0^{-1}, which becomes horizontally linearly polarized. If any axis of a quarter-wave plate makes the angle 45° with the polarization azimuth of the laser beam, then the beam at the interferometer input is circularly polarized. In this case, the use of a half-wave plate at any leg of an interferometer provides formation of orthogonally circularly polarized modes at the output.

Desirable balance of integral powers of the mixed modes LG_0^{-1} and LG_1^{-1}, in our case $1:0.45(\pm 0.03)$, is provided by the use of controllable neutral attenuators NF_1 and NF_2 at two legs of an interferometer.

One of the beams is reflected from a stationary mirror M, and another beam is reflected from a mirror PM mounted at piezoceramics (a half-wave voltage is $5.5\ V$, and the step of voltage change is $0.1\ V$ that corresponds to change of a phase difference at the legs of an interferometer $\approx 0.018\,\pi$).

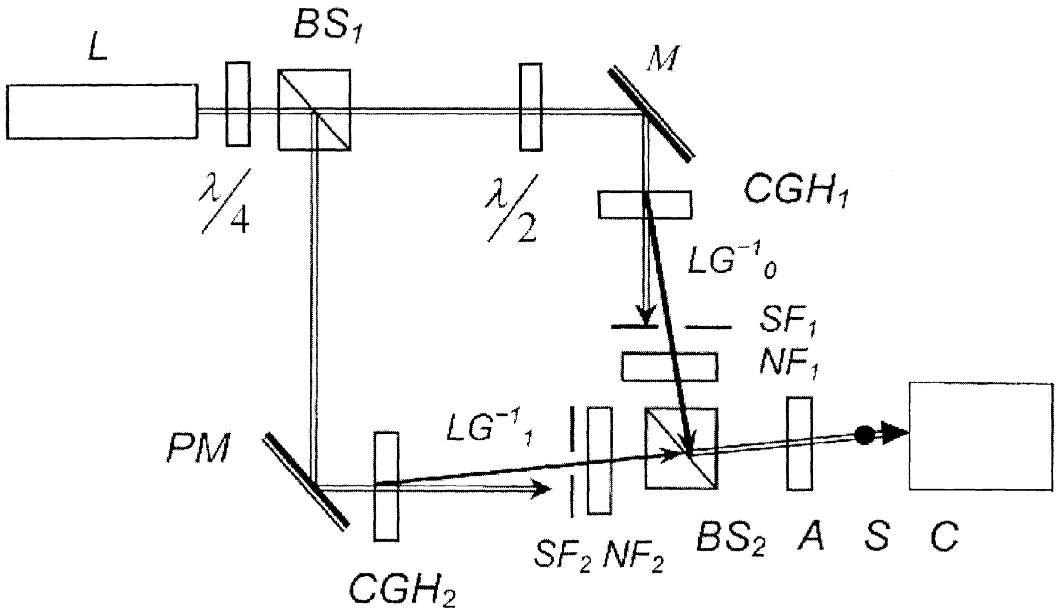

Fig. 5. Experimental arrangement: L – laser, BS_1, BS_2 - beam-splitting cubes, CGH_1, CGH_2 - computer-generated holograms for modes LG_0^{-1} and LG_1^{-1}, respectively, SF_1, SF_2 - spatial filters, NF_1, NF_2 - neutral filters, M - stationary mirror, PM - piezomirror, A - linear or circular polarization analyzer, S - diffraction screen, C - CCD-camera, $\lambda/4, \lambda/2$ - quarter- and half-wave plates.

Fine adjusting provides coaxiality of partial mixed modes at the output of an interferometer, behind a cube BS_2. Into combined beam, we place a powermeter with a pinhole (no shown in Fig. 5) for measuring of spatial distribution of intensity at the beam or (linear or circular) polarization analyzer A, i.e. the tandem of a quarter-wave plate and a linear analyzer permitting known mutual orientations of the axes of two elements. Further, symmetrically to the center of the beam we place an opaque strip S of width $d = 1\,mm$ (so that $d/2w_z \approx 0.6$) for diffraction testing of the phase structure of the beam, and CCD-camera C for registration of the experimental results.

The main experimental result is shown in Figs. 6, which corresponds to the case 2.2.1 (orthogonally linearly polarized modes LG_0^{-1} and LG_1^{-1} with $\Delta = \pi/2$). Fragments Fig. 6 *a* show the combined beam with the radial intensity distribution shown by a curve 3 at Fig. 1 that is not undergone polarization analysis, as well as the result of diffraction testing of it. One visualizes the central vortex alone, without any ring singularities, similarly to the result[4]. Namely, bending of interference fringes indicates the single-charged optical vortex with counterclockwise twirling of a phase helicoid. At the same time, the ring singularity of the field of azimuths, i.e. $C-$contour at the ring $\rho/w_z \approx 1.45$ (see Fig. 3) is not observed, in agreement with the above considerations. Fragments Fig. 6 *b* and *c* demonstrate, respectively, the results of polarization selection of the combined beam with a linear analyzer whose own polarization vector coincides with the vector of polarization of modes LG_0^{-1} or LG_1^{-1}, respectively. In Fig. 3 *c* one observes, beside of the central vortex, $L-$contour at the ring $\rho/w_z = 1$ with the constant azimuth of linear

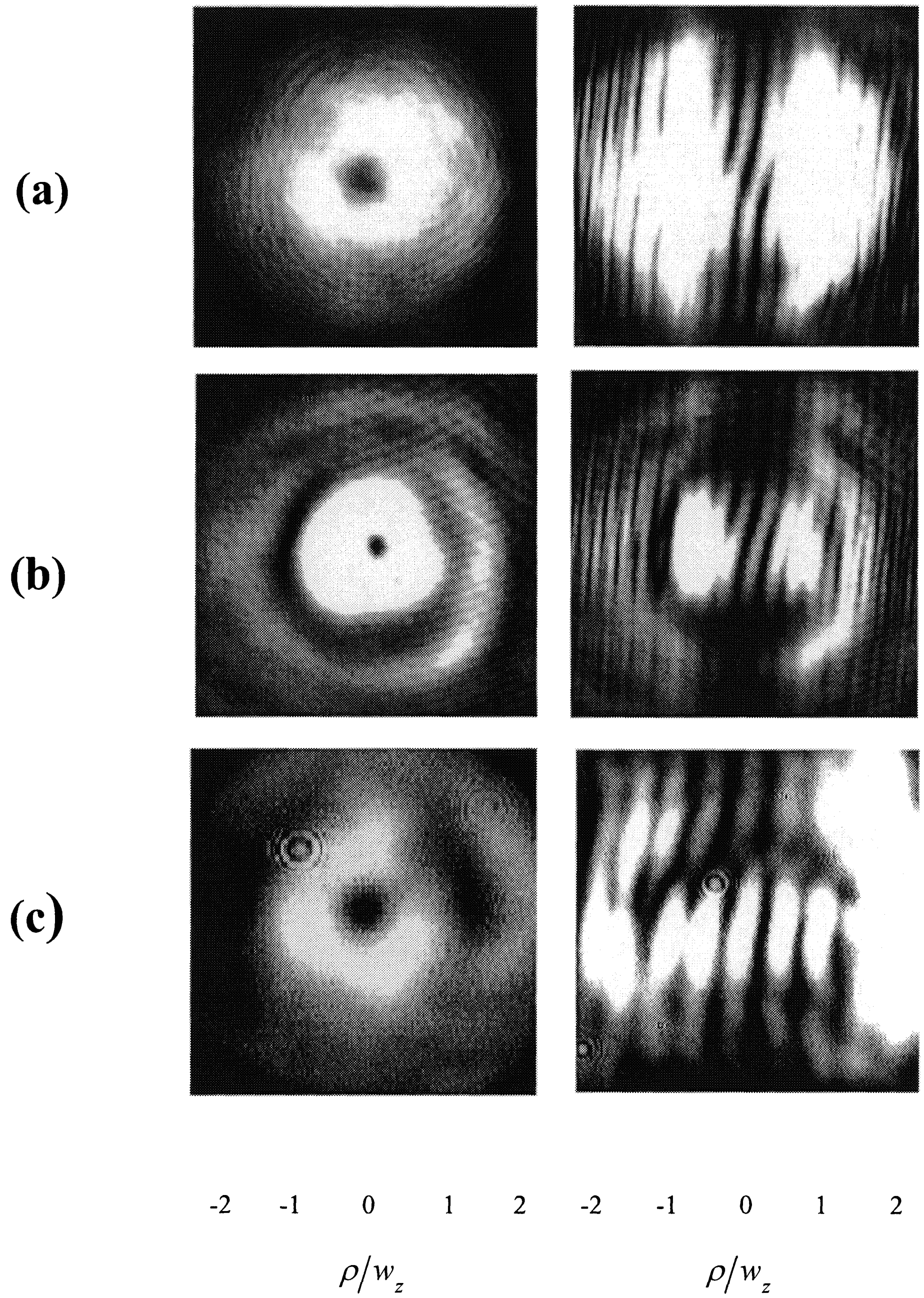

Fig. 6. Polarization structure of the combined singular beam resulting from coaxial addition of orthogonally linearly polarized modes LG_0^{-1} and LG_1^{-1} (left column – intensity distributions, right column – diffraction testing): a - combined beam without polarization selection, b - visualization of $L-$ contour at the ring $\rho/w_z = 1$ with the constant azimuth of linear polarization corresponding to the mode LG_1^{-1}, c - visualization of $C-$ contour at the ring $\rho/w_z \approx 1.45$ by polarization selection using a circle analyzer (curve 1 at Fig. 4).

polarization corresponding to the mode LG_0^{-1}. Indeed, the ring singularity at $\rho/w_z = 1$ is visualized by a half-period shift of interference fringes at inner and outer bright rings. At last, fragments Fig. 6 *d* visualize $C-$contour at the ring $\rho/w_z \approx 1.45$ by polarization selection using a circle analyzer (curve 1 at Fig. 4).

We verified experimentally that just the same result is obtained in the case, when the combined beam produced by orthogonally *circularly* polarized modes passes a *linear* polarization analyzer, and one visualize $L-$contour with the constant azimuth of linear polarization at the ring $\rho/w_z \approx 1.45$.

4. CONCLUSIONS

Summarizing, *nongeneric* polarization structures of the singular beams resulting from coaxial mixing of orthogonally polarized one-charged LG modes with the lowest mode numbers have been studied. The conditions of forming of $C-$contours (*contours of circular polarization*) and $L-$contours *with constant azimuth of linear polarization* have been determined, and typical singularities of polarization parameters into combined beams have been analyzed. Simulation results have been verified with experimental demonstrations by combination of the polarization analysis and the diffraction diagnostics of optical singularities.

Radial intensity profile of the studied beams is equal to such profile of scalar combined beams resulting from mixing of mutually incoherent modes LG_0^1 and LG_1^1 with the same ratio of the integral powers of these modes[4]. Diffraction diagnostics of the central optical vortices is the same in both cases. In polarization version, however, one can directly reveal the ring singularities of polarization parameters at the ring, where the mixed modes become equal in amplitude. Such possibility is absent in scalar case[4], where any analogue of the polarization analyzer is absent, and one can judge only indirectly on the presence of the ring singularity of the cross-spectral density of the field at the ring $\rho/w_z \approx 1.45$.

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